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AN ITERATED SEQUENTIAL TRANSPORT APPROXIMATION FOR ANISOTROPIC SPECTRAL MOMENTS*

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ABSTRACT

An improved procedure for generating anisotropic weighting spectra is derived. This procedure leads to differences from previous methods in the presence of strong resonance structure.

In transport calculations, anisotropic scattering can play a substantial role in the magnitude and shape of the predicted spectrum, particularly in the vicinity of a source, strong absorber, or a vacuum or media interface. To generate the multigroup constants required by transport codes, it is necessary to collapse pointwise data by some spectrum weighting recipe , such as,

$$\sum_{L}^{I\to J} = \frac{\int_I \int_J \sum_L (E'\to E)\phi_\ell(E')dE\, dE'}{\int_I \phi_\ell(E')\, dE'} \qquad (1)$$

Since it is the flux spectra which are to be determined by the transport calculation, a guess must be made of the flux moments to obtain group constants. Preeg (1) has demonstrated that the use of elementary weighting spectra can lead to significant errors in transport calculations. Thus it is necessary to remove as much uncertainty as possible from the initial guess of the flux moments.

The scalar flux ϕ_o may be estimated from the pointwise cross section data for finite systems by

$$\phi_{so}(E) = \frac{1}{\sum_T(E) + D(E)B^2}, \qquad (2)$$

which, for relatively narrow group widths, may be adequate. Alternatively, one may perform a fundamental mode calculation such as MC^2(2), excited by some appropriate source, and obtain scalar fluxes with the same energy resolution as the input data.

To obtain a formalism for higher ordered flux moments(3), one may take the last of the PL equations,

$$\frac{L}{2L+1}\frac{\partial \phi_{L-1}(E)}{\partial X} + \sum_T(E)\phi_\ell(E) = \int_o^\infty \sum_{SL}(E'\to E)\phi_L(E')\, dE', \qquad (3)$$

and impose the assumption that

$$\int_0^\infty \Sigma_{SL}(E' \to E)\phi_L(E')dE' \approx \Sigma_{SL}(E)\phi_L(E) \quad . \tag{4}$$

This leads to

$$\phi_L(E) = A_L \frac{\phi_{L-1}(E)}{\Sigma_T(E) - \Sigma_{SL}(E)} \tag{5}$$

where the A_L is a constant characterized by L. When this result is substituted into the preceding P_ℓ equation it is immediately apparent that a recursion relationship occurs of the same form as Equation 5 with L replaced by the more general ℓ. In each $P\ell$ equation, it is necessary to impose the approximation of Equation 4 appropriate to its respective order. This process, known as the "sequential transport approximation" has been used by Ginsberg and Becker (4) and incorporated into an RPI version of the SUPERTOG code (5).

Since errors introduced from this approximation will propagate to higher ordered moments, a process which will eliminate the approximation may be of value, particularly where high orders of anisotropy are being considered. The "iterated sequential transport approximation", which we introduce in this note, removes this propagating error by a straightforward ,simple process.

This method consists basically of a repeated application of Equations (4) and (5) to obtain a converged flux moment. E.g., for the first flux moment, one uses Equation 5 to obtain a first

guess of $\phi_1(E)$ which we denote by $\phi_1^{(1)}(E)$. This value is then substituted into Equation 4 to obtain a first guess on $\Sigma_{s1}(E)$ by

$$\sum_{s1}^{(1)}(E) = \frac{\int_0^\infty \sum_{s1}(E' \to E)\, \phi_1^{(1)}(E')\, dE'}{\phi_1^{(1)}(E)} \qquad (6)$$

This first guess on $\Sigma_{s1}^{(1)}(E)$ is then substituted back into Equation 5 to obtain $\phi_1^{(2)}(E)$, and the process is continued until convergence is obtained. The converged $\phi_1(E)$ is then used to obtain a converged $\phi_2(E)$ and so on to whatever degree of anisotropy desired.

The ordinary and iterated sequential transport approximations have been applied to collapsing multigroup constants to few group constants. Table 1 provides a comparison of results for a four-group P_3 approximation in iron. It may be observed that results for the two approximations are quite similar except for the fourth group, where some significant differences are observed. These differences are attributed to the strong resonance structure of the cross sections within group four. We therefore conclude that the ordinary sequential transport approximation should be adequate for many purposes, but that the iterated method may be necessary when strong resonance structure is involved.

TABLE I

COMPARISON OF GROUP CONSTANTS* OBTAINED FROM THE SEQUENTIAL AND INTERATED SEQUENTIAL TRANSPORT APPROXIMATIONS

Order of Scattering k	Energy Group	Sequential Transport Approximation			Iterated Sequential Transport Approximation		
		$\Sigma_t^g(K)$**	$\Sigma_k^{g\to g}$	$\Sigma_k^{g+1\to g}$	$\Sigma_t^g(K)$	$\Sigma_k^{g\to g}$	$\Sigma_k^{g+1\to g}$
1	1	3.078	.8295	0.	3.076	.8272	0.
1	2	2.751	.4769	-.03089	2.754	.4763	-.03096
1	3	2.219	.1848	-.02496	2.210	.1837	-.02479
1	4	1.0366	.01402	-.01643	1.284	.01747	-.01631
2	1	3.059	.6780	0.	3.058	.6791	0.
2	2	2.618	.3014	.01256	2.620	.3014	.01253
2	3	1.056	.02610	-.00551	1.050	.02576	-.00551
2	4	.3214	.00035	.00011	.4125	.00073	.00011
3	1	3.039	.3660	0.	3.037	.3643	0.
3	2	2.515	.04614	-.00836	2.516	.04590	-.00841
3	3	.5861	.00397	-.00296	.5870	.00396	-.00297
3	4	.2346	-.00005	.00005	.2723	.00024	.00005

* Group Boundaries (in MeV) are 10.0, 1.353, .302, .0674, .01.

** $\Sigma_t^g(k)$ is the total cross-section obtained by averaging by the kth Legendre Moment of the Weighting Spectrum.

REFERENCES

1. W. Preeg, "Group Averaging Using SUPERTOG With Arbitrary Flux Weighting," in "A Review of Multigroup Nuclear Cross-Section Preparation-Theory, Techniques, and Computer Codes," D. K. Trubey and J. Gurney, Eds., ORNL-RSIC-27, Oak Ridge National Laboratory (1969).

2. B. J. Toppel, A. L. Rags, D. M. O'Shea, "MC^2, A Code to Calculate Multigroup Cross Sections," ANL-7318 (June, 1967).

3. G. I. Bell, G. D. Hansen, H. A. Sandmeier, "Multitable Treatments of Anisotropic Scattering in S_N Multigroup Transport Calculations," Nucl. Sci. & Eng., 28, 376-383 (1967).

4. A. Ginsberg and M. Becker, "Models for Anisotropic Weighting Spectra," Trans. Am. Nucl. Soc., 14, 368 (1971).

5. R. Q. Wright, et. al., "SUPERTOG: A Program to Generate Fine Group Constants and Pn Scattering Matrices from ENDF/B," ORNL-TM-2679, Oak Ridge National Laboratory, 1969.

Footnote a:

Some authors have referred to this process as an extended transport approximation. Because of the use of the term extended to denote approximations for incorporating transport effects into transport codes (as in Reference 3), the authors prefer the term sequential to denote the approximation associated with weighting spectra.